Chapter 4: Quantum Mechanics in 3D

Quantum Mechanics

1 Angular Momentum

Definition 1.1 (Angular Momentum Operator)**.** [def:angular-momentum] The *angular momentum operator* is

$$\hat{\vec{L}} := \hat{\vec{r}} \times \hat{\vec{p}},$$

with components

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y, \quad \hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z, \quad \hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x.$$

2 Atomic Eigenstates

Definition 2.1 (Atomic Eigenstate)**.** [def:atomic-eigenstate] An *atomic eigenstate* $|\psi_{n\ell m}\rangle$ (also written $|n, \ell, m\rangle$) is the simultaneous eigenstate of $\hat{H}$, $\hat{L}^2$, and $\hat{L}_z$ with eigenvalues E_n, $\ell(\ell+1)\hbar^2$, and $m\hbar$ respectively. Its position-space wavefunction separates as

$$\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r)\, Y_\ell^m(\theta, \phi),$$

where $R_{n\ell}$ is the radial function and Y_ℓ^m is the spherical harmonic.

Theorem 2.1 (Parity of Spherical Harmonics)**.** [thm:spherical-harmonic-parity] *Under* $(\theta, \phi) \to (\pi - \theta,\, \phi + \pi)$,

$$Y_\ell^m(\pi - \theta,\, \phi + \pi) = (-1)^\ell\, Y_\ell^m(\theta, \phi).$$

Proof. **TODO** □